mentum  A library of solution is made ahead of time for the possible combinations of shock and rarefactions that would be possible between two adjacent zones

The method is applied to a single fluid with an equation of state where the pressure is a function of density alone  The technique could be readily extended to a two-fluid problem  However, to use an equation of state that had an energy dependence would add a considerable complication to the calculation

In general the method appears to be a practical way of constantly rezoning a hydrodynamic calculation in Eulerion coordinates  It is especially suited to the problem of a gas expanding into a vacuum  In principal the technique could also be applied to two-dimensional calculations

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# Free Molecular Heat Transfer near a Wall

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In heat transfer between a gas and a wall, the heat flux can be determined by considering the transfer of heat at the wall through a layer so thin that the chances of intermolecular collisions can be disregarded

Blodgett and Langmuir[1] have analyzed heat transfer between two coaxial cylinders under the following conditions  Heat from the inside cylinder at a temperature $T$ is transferred by "free" molecules over a distance equal to the mean free path $l$, where the temperature is $T'_a$ (the difference $T - T_a'$ is assumed to be the temperature drop)  The heat is then transferred in accordance with the usual laws of heat conduction

Without imposing limitations due to the possible effect of curvature of the cylinder, the investigators arrived at the following equation for the heat flux:

$$g = a\left(\beta + \frac{1}{2}\right) pk \frac{1}{\sqrt{2\pi mkT'_a}} (T - T_a') \tag{1}$$

where $a$ is the accommodation coefficient on the surface of the inside cylinder, $p$ the pressure, $k$ the Boltzmann constant, $m$ the mass of the molecule, and $\beta k$ the specific heat per molecule at constant volume, in erg/K

The number of molecules incident on a surface area of 1 cm$^2$/sec is

$$N = \tfrac{1}{4} n\bar{v} \tag{2}$$

where $n$ is the number of molecules per unit volume, and $v$ is their arithmetic mean velocity

Accordingly, we can transform Eq  (1) as follows:

$$g = a\Lambda_0 p \sqrt{273\ 2/T_a}(T - T_a') \tag{3}$$

where $\Lambda_0$ is the thermal conductivity of the "free" molecules in erg/sec cm$^2$ deg bar:

$$\Lambda_0 = \frac{1}{2}\left(\frac{\gamma + 1}{\gamma - 1}\right)\sqrt{\frac{R}{2\pi}} \frac{1}{\sqrt{M(273\ 2)}} \tag{4}$$

where $\gamma = c_p/c$  is the ratio of heat capacities

Kyte, Madden, and Piret[2] employ Eq  (3) in determining the limits of the effect of free molecular transfer on heat exchange at low pressures, and also in calculating the heat flux

Deshman[3] obtained an equation of type (3) for free molecular transfer between two infinite plates  Instead of $a$ he used a reduced accommodation coefficient

$$a_{\text{du d}} = a_1 a_2/(a_1 + a_2 - a_1 a_2) \tag{5}$$

where $a_1$ and $a_2$ are the accommodation coefficients on the surface of the plates

Equation (3) represents a case of heat transfer between plates in which the accommodation coefficient on one of the plates is equal to unity  This means that if Eq  (3) is to remain valid for the given heat-transfer conditions, we must assume the accommodation coefficient to be equal to unity on the gas side

Blodgett and Langmuir do in fact make this assumption in their physical model, assuming that the temperature of the molecules incident on the surface is $T_a$  But $T_a$ is the average temperature of the molecules incident on the wall from the gas side and moving away from the wall  The number of molecules with opposite velocity components is equal; consequently, molecules at a temperature higher than $T_a$ are incident on the wall  Hence, the heat flux should not be expressed by Eq  (3)

In our analysis we will assume that the temperature drop at the surface is $T - T_a$, where $T$ is the surface temperature and $T_a$ (contrary to the premises adopted in Ref  1) is the temperature of the gas adjacent to the wall  This corresponds to the formulation of the boundary conditions in problems of rarefied gas mechanics

Let us compare the free molecular heat transfer near a plane horizontal wall with the heat conduction far from it

Let us select a reference surface far from the wall  Let the temperature of the gas on both sides of the selected plane, at a distance of the mean free path from the plane, be $T''$ and $T'$, respectively  It is also assumed that $T'' > T'$

The heat flux through the reference surface is

$$g = -\lambda\Delta T/\Delta x \tag{6}$$

where $\lambda$ is the thermal conductivity, $\Delta T/\Delta x$ is the temperature gradient, which can usually be expressed[4] in the following terms:

$$\Delta T/\Delta x = (T'' - T')/2\bar{l} \tag{7}$$

When the difference $T'' - T'$ is small, the number of molecules moving up through the reference plane is equal to the number of molecules moving down through it  This is because the energy of the molecules incident on the plane from both sides is equally and incrementally higher in absolute

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Translated from Inzhenerno-Fizicheskii Zhurnal (Journal of Engineering Physics) 5, no 1, 111–114 (1962)  Translated by Scripta Technica, Inc , New York terms than the energy corresponding to the temperature of the reference plane Consequently, one half of the heat flux is due to the upward motion of the molecules, and the other half is due to the downward motion

Let us examine heat transfer near a solid wall The molecules moving toward it from a distance approximately equal to $\bar{l}$, transfer the same excess energy as those moving across the reference plane on one side, i e , one half of the total heat flux In steady state, the heat flux is the same at any distance from the wall This means that the second half of the heat flux near the wall should be contributed by the receding molecules This is only possible if there is a temperature difference between the receding molecules and the average temperature of the gas at the wall, or, in other words, if there is a temperature gradient

Having left the wall, the molecules transfer their excess energy to the gas This excess energy expressed per unit area of 1 cm$^2$/sec is equal to

$$\Delta E = \frac{\gamma + 1}{8} \rho \bar{v} c \, (T - T_a) \qquad (8)$$

where $\rho$ is the density of the gas

The heat equilibrium equation at the wall for total energy exchange ($\alpha = 1$) can be written in the form

$$g/2 + \Delta E = g \qquad (9)$$

But under actual conditions $a < 1$ Accordingly, the equilibrium equation takes the form

$$a(g/2 + \Delta E) = g \qquad (10)$$

whence

$$g = 2a\Delta E/(2 - a) \qquad (11)$$

Equation (8) can be reduced to

$$\Delta E = \Lambda_0 p \sqrt{273\ 2/T_a}(T - T_a) \qquad (12)$$

Substituting this value into (11), we find the following expression for the heat flux at the wall

$$g = 2a\Lambda_0 p \sqrt{273\ 2/T_a}(T - T_a)/(2 - a) \qquad (13)$$

Let us express the temperature difference by

$$\Delta T = \frac{2 - a}{2a} \, \frac{g}{\Lambda_0 p \sqrt{273\ 2/T_a}} \qquad (14)$$

The foregoing equation is similar to the well-known and widely used Kennard equation

$$\Delta T = \frac{2 - a}{2a} \, \frac{4\gamma}{\gamma + 1} \, \frac{\bar{l}}{Pr} \, \frac{dt}{dx} \qquad (15)$$

where $dt/dx$ is the temperature gradient at the wall

Let us compare Eqs (3) and (13) The temperature $T_a'$ can be considered equal to $T_a$ provided we further assume that there are no intermolecular collisions at a distance from the wall equal to the mean free path In this case $g$, as calculated from (13), proves to be greater than in Eq (3) by a factor of $2/(2 - a)$

Since Eq (13) corresponds to universal temperature boundary conditions, it should be given preference in calculations, particularly when the accommodation factor is close to unity

### References

[1] Blodgett, K B and Langmuir, I , Phys Rev **40,** 78–104 (1932)

[2] Kyte, J , Madden, A , and Piret, E , Chem Eng Prog **49,** 12 (1953)

[3] Deshman, S , *Scientific Fundamentals of Vacuum Engineering* (Foreign Literature Press, Moscow, 1950)

[4] Frish, S E and Timoreva, A B , *Kurs fiziki* (*Course in Physics*) (Moscow, 1959), Vol 1

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# Digest of Translated Russian Literature

**The following abstracts have been selected by the Editor from translated Russian journals supplied by the indicated societies and organizations, whose cooperation is gratefully acknowledged Information concerning subscriptions to the publications may be obtained from these societies and organizations Note: Volumes and numbers given are those of the English translations, not of the original Russian**

## *INDUSTRIAL LABORATORY* (*Zavodskaia Laboratoriia*) Published by Instrument Society of America, Pittsburgh, Pa

### Volume 28, Number 1, January 1962

**Potentiometric Method for Analyzing Mixtures of Organic Acids With Nitric Acid in Nonaqueous Media,** A P Kreshkov L N Bykova, M S Rusakova, and N A Kazaryan, pp 10–12

The method is based on the titration of a mixture of nitric and organic acids, in methyl ethyl ketone with 0 1 N tetraethylammonium hydroxide in benzene-methanol or in acetone with 0 1 N sodium hydroxide in aqueous acetone It is used for analyzing production mixtures of nitric and $\alpha$-hydroxyisobutyric acids

**Measurement of Absolute Viscosity of a Liquid by the Oscillation Method,** G S Rosin, pp 72–74

An oscillation method has been developed for measuring the absolute viscosity A flat vane is used with this method as the oscillating member An equation has been derived for the calculation of the resistance force and the dynamic viscosity from the data obtained by the resonance method

**Method for Investigation of Fatigue Failure of Metals by Measuring the Variation of Losses Caused by Magnetic Polarity Reversals,** O I Gushcha, pp 76–79

A method and apparatus are suggested for the investigation of the fatigue failure of metals by measuring the change of the magnetic polarity reversal losses in the specimen; they make possible the detection of the place where the fatigue is localized